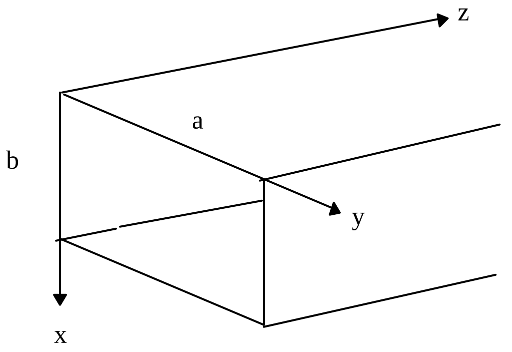
## 8.2 Ableitung der Felder für die Grundmode im Rechteckhohlleiter

Die Felder für die $TE_{10}$ -Mode im Rechteckhohleiter sollen nun direkt aus den Maxwell'schen Gleichungen unter den gegebenen Randbedingungen bestimmt werden.




$$\vec{E}(x,y,z,t) = \vec{E}(x,y,z)e^{j\omega t}, \vec{H}(x,y,z,t) = \vec{H}(x,y,z)e^{j\omega t}$$

$$rot\vec{E} = -j\omega\mu_0\vec{H}$$

$$rot\vec{H} = j\omega\varepsilon_0\vec{E}$$

einfachster Ansatz : $\vec{E}(x,y,z) = E_x(y,z)\vec{e}_x$ also $\frac{\partial\ldots}{\partial x} = 0$

$$rot\vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & \frac{\partial\ldots}{\partial y} & \frac{\partial\ldots}{\partial z} \\ & & \end{vmatrix} = \vec{e}_y\frac{\partial E_x}{\partial z} - \vec{e}_z\frac{\partial E_x}{\partial y} = -j\omega\mu_0(H_y\vec{e}_y + H_z\vec{e}_z)$$

$$rot\vec{H} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & \frac{\partial\ldots}{\partial y} & \frac{\partial\ldots}{\partial z} \\ & H_y & H_z \end{vmatrix} = \vec{e}_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = j\omega\varepsilon_0 E_x\vec{e}_x \quad \Rightarrow$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu_0 H_y$$

$$\frac{\partial E_x}{\partial y} = j\omega\mu_0 H_z$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon_0 E_x \quad \Rightarrow$$

$$\frac{1}{j\omega\mu_0}\left(\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) = j\omega\varepsilon_0 E_x \quad \Rightarrow$$

Wellengleichung für E:

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu_0 \varepsilon_0 E_x = 0$$

Lösungsansatz:

$$\boxed{\begin{array}{l} E_x(y,z) = \hat{E}\sin\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z} \\ \beta_0 = \frac{2\pi}{\lambda_0} \quad \beta_H = \frac{2\pi}{\lambda_H} \end{array}}$$

Einsetzen in Wellengleichung :

$$-\left(\frac{\pi}{a}\right)^2 + \left(-j\beta_H\right)^2 + \left(\frac{\omega}{c_0}\right)^2 = 0 \qquad \frac{\omega}{c_0} = \beta_0$$

$$\beta_H{}^2 = \beta_0{}^2 - \left(\frac{\pi}{a}\right)^2 \qquad \frac{2\pi}{\lambda_H} = \sqrt{\left(\frac{2\pi}{\lambda_0}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

$$\frac{1}{\lambda_H} = \frac{1}{\lambda_0}\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}$$

$$\boxed{\lambda_H = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}} \qquad \lambda_{gr} = 2a}$$

H-Feld : E einsetzen in Induktionsgesetz (1. Maxwell‘sche Gl. s.o.)

$$E_x(y,z) = \hat{E}\sin\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z}$$

$$-j\beta_H \hat{E}\sin\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z} = -j\omega\mu_0 H_y$$

$$H_y = \hat{E}\frac{\beta_H}{\omega\mu_0}\sin\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z}$$

$$\boxed{H_y = Z_H \hat{E}\sin\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z} \qquad \text{mit} \qquad Z_H = \frac{Z_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}}$$

$$\hat{E}\left(\frac{\pi}{a}\right)\cos\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z} = j\omega\mu_0 H_z$$

$$H_z = -j\frac{\hat{E}}{\omega\mu_0}\left(\frac{\pi}{a}\right)\cos\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z}$$

$$\boxed{H_z = -j\frac{\hat{E}}{Z_0}\frac{\lambda_0}{2a}\cos\left(\frac{\pi}{a}y\right)e^{-j\,ß_H z}}$$

$$E_x(y,z) = \hat{E}\sin\left(\frac{\pi}{a}y\right)e^{-j\beta_H z}$$

$$-j\beta_H\hat{E}\sin\left(\frac{\pi}{a}y\right)e^{-j\beta_H z} = -j\omega\mu_0 H_y$$

$$H_y = \hat{E}\frac{\beta_H}{\omega\mu_0}\sin\left(\frac{\pi}{a}y\right)e^{-j\beta_H z}$$

$$H_y = Z_H\hat{E}\sin\left(\frac{\pi}{a}y\right)e^{-j\beta_H z} \qquad \text{mit} \qquad Z_H = \frac{Z_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}$$

$$\hat{E}\left(\frac{\pi}{a}\right)\cos\left(\frac{\pi}{a}y\right)e^{-j\beta_H z} = j\omega\mu_0 H_z$$

$$H_z = -j\frac{\hat{E}}{\omega\mu_0}\left(\frac{\pi}{a}\right)\cos\left(\frac{\pi}{a}y\right)e^{-j\beta_H z}$$

$$H_z = -j\frac{\hat{E}}{Z_0}\frac{\lambda_0}{2a}\cos\left(\frac{\pi}{a}y\right)e^{-j\beta_H z}$$

die folgenden Bilder sind aus Sinnema [11] entnommen:

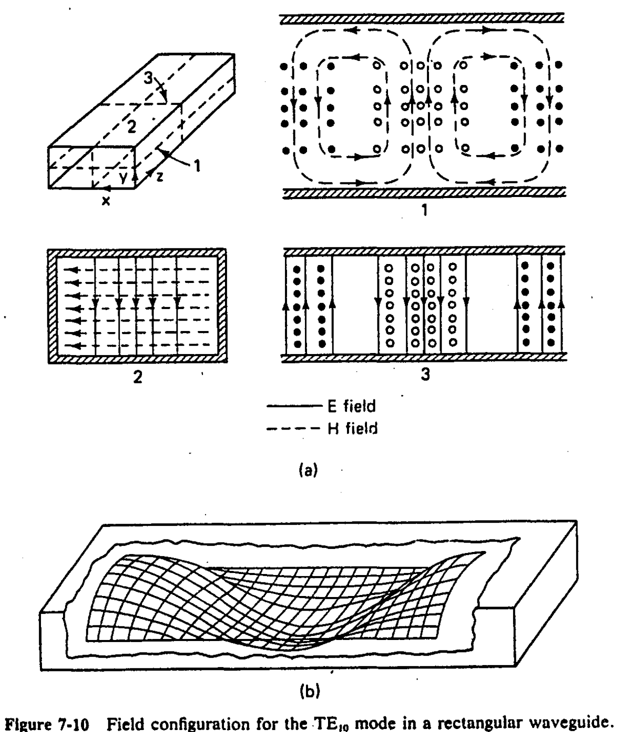



**Figure 7-10** Field configuration for the $TE_{10}$ mode in a rectangular waveguide. (a) Closeness of line spacing indicates the strength of the field. Dots represent field lines coming out of the plane of the paper. Small circles represent lines going into paper. [From S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Wves in Communication Electronics* (New York: John Wiley & Sons, Inc., 1965), Table 8.02, p. 423.] (b) Three-dimensional view of the electric field.