# BENDING STRENGTH OF GLULAM BEAMS

## - A STATISTICAL MODEL -

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Paper presented at the
IUFRO Wood Engineering Group Meeting
Saint John/New Brunswick, Canada
July/August 1990

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## - A statistical model -

by
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## 1 General

The scope of a current research program[1] is the investigation of bending strength of glulam beams with regard to the developement of design proposals. The "Karlsruhe calculation model" (Ehlbeck et al. 1985, Colling 1988) - a finite element model which calculates the strength of glulam beams by means of Monte Carlo simulations - was thought to achieve this purpose.

The simulations however showed that the strength of glulam beams is a very complex field and that it is very difficult to describe the influence of one single parameter. The "Karlsruhe calculation model" does correctly compute every possible tendency, but the problem is to describe these tendencies mathematically.

## 2 A statistical model

### 2.1 Splitting up the final product glulam into the two "materials" wood and finger joint

Colling 1988 pointed out that the strength of glulam beams is affected by two factors - the strength of wood (including knots) and the strength of finger joints - and that the outcome of a grading method depends on the balance of these two factors.

Based on these results, a statistical model was developed (Colling 1990a), which sets up the following definitions:

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[1] Ehlbeck, J.; Colling, F.: Biegefestigkeit von Brettschichtholzträgern in Abhängigkeit von den Eigenschaften der Brettlamellen im Hinblick auf Normungsvorschläge.

- glulam consists of two "materials" wood and finger joint. But here the "material" finger joint does not stand for the joint itself, but for the glulam beams, which fail due to a finger joint failure. In analogy to this, "material" wood stands for those beams that fail in the area of wood defects (knots);
- the "true" strength distribution of "material" finger joint may be obtained by tests on condition that all beams fail due to a finger joint failure. In analogy to this, the "true" strength distribution of "material" wood correspondents to that distribution determined by tests supposing that all beams fail in the area of knots.

On the basis of these "true" strength distributions of the two "materials", the statistical model makes it possible to calculate the strength characteristics of the final product glulam.

Now the legitimate question arises how to determine the "true" strength distributions of the two "materials", because in most cases, both failure modes do occur during bending tests with glulam beams.

In case of "material" wood the corresponding "true" strength distributions might be determined with beams having no finger joints at all. But this possibility is limited to the case of small beams, because large glulam beams can only be produced with finger jointed laminations and thus a finger joint failure can not be excluded.

In case of "material" finger joint, the "true" strength distributions can only be determined with glulam beams having high strength laminations and low strength finger joints at the same time. But as the strength of both wood and finger joints increase with increasing wood properties (density, MOE), the experimental determination of the required strength distribution is hardly possible.

Within the "Karlsruhe calculation model", however, it is possible to control artifically the strength of finger joints as well as wood quality so that a desired failure mode can be forced. Hence the required "true" strength distributions of the two "materials" wood and finger joint may be determined with the help of the "Karlsruhe calculation model".

## 2.2 Development of the model

A glulam beam has got two possibilities of failure (knot and finger joint) and in order to escape loading it will choose the failure mode resulting in the lower strength value. The probability of a glulam beam surviving a given stress $\sigma$ is therefore equal to the probability that neither a wood failure nor a finger joint failure occur. This probability of survival may be written:

$$1 - H(\sigma) = (1-F(\sigma)) \cdot (1 - G(\sigma)) \tag{1}$$

where

$H(\sigma)$ = probability that a glulam beam fails at a given stress $\sigma$,
$F(\sigma)$ = probability that "material" finger joint fails at a given stress $\sigma$,
$G(\sigma)$ = probability that "material" wood fails at a given stress $\sigma$.

F and G are equal to the "true" distribution functions of the two "materials". H corresponds to the resulting strength distribution of the final product glulam.

*Eq.(1)* may also be written as:

$$H(\sigma) = F(\sigma) + G(\sigma) - F(\sigma) \cdot G(\sigma) \tag{2}$$

In the further course of this paper, the distribution functions F and G are assumed to be normal distributed (Gauss) with the corresponding mean values $m_1$ and $m_2$, the coefficients of variation $v_1$ and $v_2$ and the corresponding 5th-percentiles $x_{5,1}$ and $x_{5,2}$. Hence, *eq(2)* results in

$$H(\sigma) = \Phi\left(\frac{\sigma/m_1-1}{v_1}\right) + \Phi\left(\frac{\sigma/m_2-1}{v_2}\right) - \Phi\left(\frac{\sigma/m_1-1}{v_1}\right) \cdot \Phi\left(\frac{\sigma/m_2-1}{v_2}\right) , \tag{3}$$

where $\Phi$ is the distribution function of the standardized Gauss-distribution with mean value 0 and standard deviation 1.

The strength of glulam beams is mainly governed by the weaker "material": from *eq(1)* follows that a percentile of the glulam strength distribution H can at most take the corresponding value of the weaker "material" F. This tendency is shown in *fig. 1*. In the extreme case, there is only one failure mode, and the strength distribution of glulam beams is identical with the "true" strength distribution of the weaker "material". In the further course of this paper, the weaker "material" is defined as the material with the lower 5th-percentile $x_{5,1}$.

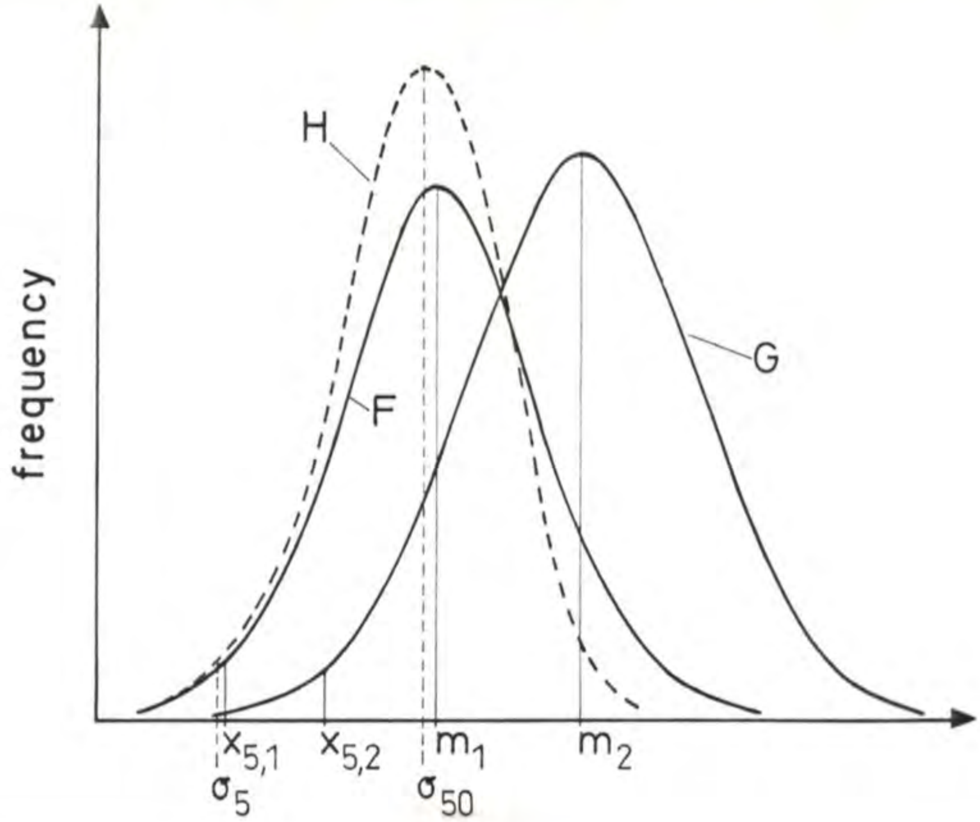



*fig. 1:* Strength distributions F, G and H

According to *eq(3)*, the characteristic bending strength $\sigma_5$ of glulam beams (5th-percentile) may be determined by

$$0{,}05 = \Phi\left(\frac{\sigma_5/m_1-1}{v_1}\right) + \Phi\left(\frac{\sigma_5/m_2-1}{v_2}\right) - \Phi\left(\frac{\sigma_5/m_1-1}{v_1}\right) \cdot \Phi\left(\frac{\sigma_5/m_2-1}{v_2}\right) \quad . \qquad (4)$$

The median value $\sigma_{50}$ of the bending strength distribution (which is not necessarily normally distributed) may be calculated by:

$$0{,}5 = \Phi\left(\frac{\sigma_{50}/m_1-1}{v_1}\right) + \Phi\left(\frac{\sigma_{50}/m_2-1}{v_2}\right) - \Phi\left(\frac{\sigma_{50}/m_1-1}{v_1}\right) \cdot \Phi\left(\frac{\sigma_{50}/m_2-1}{v_2}\right) \qquad (5)$$

As the strength distribution of glulam beams depends on the distance and the overlap between the distributions of the two "materials", it is essential to describe appropriately the relation between these distributions. For that purpose, the ratios $x_{5,1}/x_{5,2}$ of the 5th-percentiles and $m_1/m_2$ of the mean values, as well as the coefficient of variation $v_1$ were chosen. Furthermore the strength values $\sigma_5$ and $\sigma_{50}$ of glulam were referred to the corresponding strength values $x_{5,1}$ and $m_1$ of the weaker "material".

With the help of the following equations

$$x_{5,1} = m_1 \cdot (1 - 1{,}645 \cdot v_1) \qquad (6a)$$

and

$$x_{5,2} = m_2 \cdot (1 - 1{,}645 \cdot v_2) \qquad (6b)$$

*eq(4)* and *eq(5)* may after some transformations be written as:

$$0{,}05 = \Phi\left(\frac{(1-1{,}645\cdot v_1)\cdot \sigma_5/x_{5,1} - 1}{v_1}\right) + \Phi\left(1{,}645 \cdot \frac{(1-1{,}645\cdot v_1)\cdot \sigma_5/x_{5,1}\cdot m_1/m_2 - 1}{1 - \frac{m_1/m_2}{x_{5,1}/x_{5,2}}\cdot(1-1{,}645\cdot v_1)}\right)$$

$$-\Phi\left(\frac{(1-1{,}645\cdot v_1)\cdot \sigma_5/x_{5,1} - 1}{v_1}\right) \cdot \Phi\left(1{,}645 \cdot \frac{(1-1{,}645\cdot v_1)\cdot \sigma_5/x_{5,1}\cdot m_1/m_2 - 1}{1 - \frac{m_1/m_2}{x_{5,1}/x_{5,2}}\cdot(1-1{,}645\cdot v_1)}\right)$$

(7)

and

$$0{,}5 = \Phi\left(\frac{\sigma_{50}/m_1 - 1}{v_1}\right) + \Phi\left(1{,}645\cdot\frac{\sigma_{50}/m_1\cdot m_1/m_2 - 1}{1 - \dfrac{m_1/m_2}{x_{5,1}/x_{5,2}}\cdot(1-1{,}645\cdot v_1)}\right)$$

$$-\Phi\left(\frac{\sigma_{50}/m_1 - 1}{v_1}\right)\cdot\Phi\left(1{,}645\cdot\frac{\sigma_{50}/m_1\cdot m_1/m_2 - 1}{1 - \dfrac{m_1/m_2}{x_{5,1}/x_{5,2}}\cdot(1-1{,}645\cdot v_1)}\right) \qquad (8)$$

The courses of the ratios $\sigma_5/x_{5,1}$ and $\sigma_{50}/m_1$ depending on the parameters $x_{5,1}/x_{5,2}$ and $m_1/m_2$ are shown in *fig. 2* and *fig. 3* for the case of $v_1 = 0{,}20$.

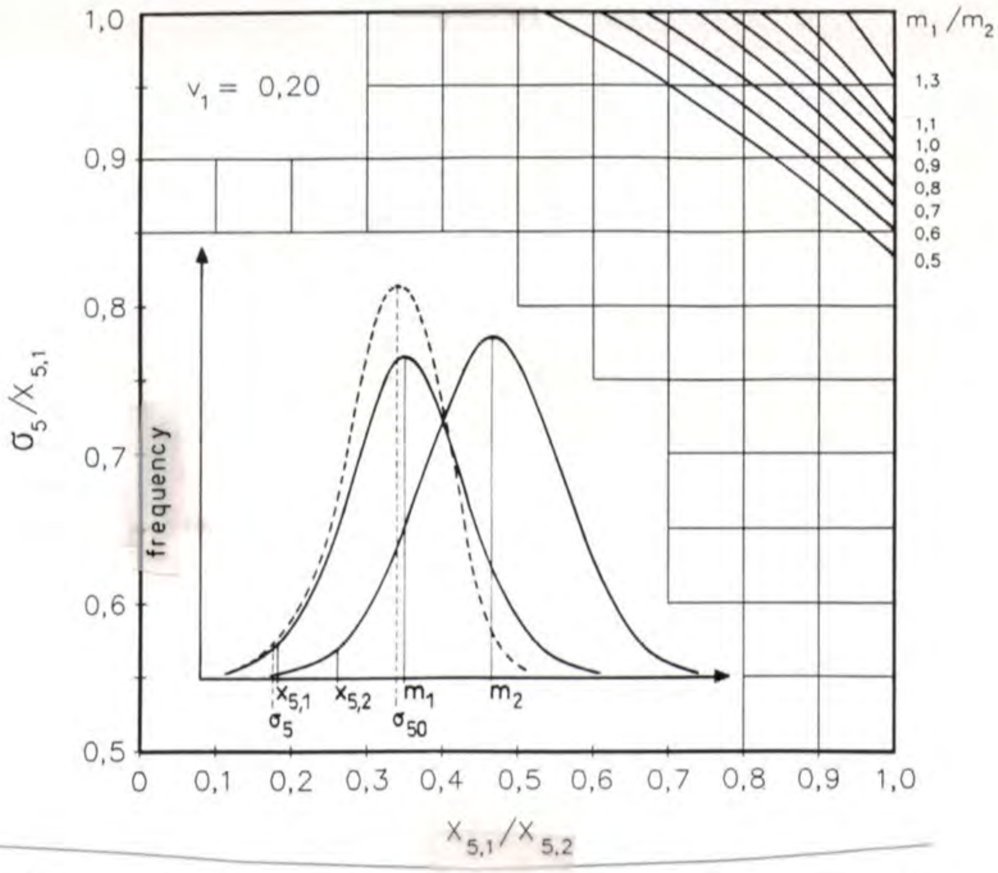



*fig. 2:* Ratio $\sigma_5/x_{5,1}$ depending on $x_{5,1}/x_{5,2}$ and $m_1/m_2$ ; $v_1 = 0{,}2$

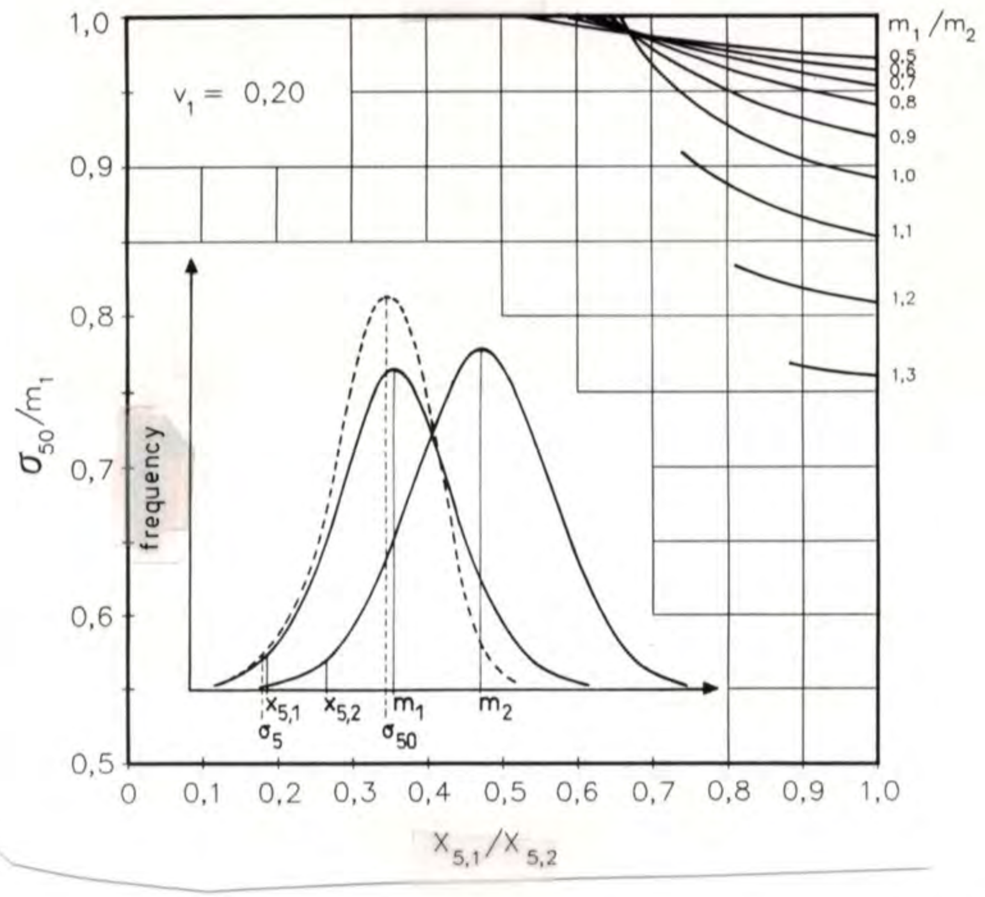



*fig. 3:* Ratio $\sigma_{50}/m_1$ depending on $x_{5,1}/x_{5,2}$ and $m_1/m_2$ ; $v_1 = 0{,}2$

Furthermore, it is possible to estimate how often the "material" noted 2 will be responsible for the beam failure. The frequency of occurence $P_2$ may be calculated in analogy to the probability of failure within a probabilistic safety concept according to Steck 1982:

$$P_2 = \Phi \left( \frac{m_1 - m_2}{\sqrt{s_1^2 + s_2^2}} \right) \quad . \tag{9}$$

where

$\Phi$ = distribution function of the standardized normal distribution,

$m_1, m_2$ = mean values of "materials" 1 and 2,

$s_1, s_2$ = standard deviations of "materials" 1 and 2.

## 2.3 Numerical example

The practical use of the statistical model shall now be demonstrated by a numerical example.

Given are the normal distributed "true" strength distributions of the "materials" wood and finger joint (fj) with the mean values $m_{wood} = 40$ N/mm$^2$ and $m_{fj} = 36$ N/mm$^2$ and the corresponding standard deviations $s_{wood} = 6{,}0$ N/mm$^2$ and $s_{fj} = 7{,}2$ N/mm$^2$.

The 5th-percentiles may be calculated according to *eq(6)*:

$$x_{5,wood} = 40 \cdot (1 - 1{,}645 \cdot \frac{6{,}0}{40}) = 30{,}1 \text{ N/mm}^2$$

$$x_{5,fj} = 36 \cdot (1 - 1{,}645 \cdot \frac{7{,}2}{36}) = 24{,}2 \text{ N/mm}^2 \quad .$$

"Material" finger joint is found to be the weaker "material" with the lower 5th-percentile, and the entry data of the statistical model are computed to be:

$$x_{5,1}/x_{5,2} = 24{,}2/30{,}1 = 0{,}80$$

$$m_1/m_2 = 36/40 = 0{,}90$$

$$v_1 = 7{,}2/36 = 0{,}20 \quad .$$

The expected 5th-percentile $\sigma_5$ and the median value $\sigma_{50}$ of glulam beams may be obtained from *fig. 2* and *fig. 3*:

$$\sigma_5 = 0{,}99 \cdot 24{,}2 = 24{,}0 \text{ N/mm}^2$$

$$\sigma_{50} = 0{,}95 \cdot 36 = 34{,}2 \text{ N/mm}^2 \quad .$$

The probability of a wood failure is:

$$\Phi \left(\frac{36 - 40}{\sqrt{6{,}8^2 + 9^2}}\right) = \Phi(-0{,}355) = 0{,}36 \ ,$$

that means, that only 1/3 of the beams are expected to fail in the area of knots.

This example shows that both, 5th-percentile and median value of glulam beams, orientate themselves very strongly by the corresponding values of the weaker "material".

## 2.4 Discussion

Now, what is the use of this statistical model?

The splitting up of the final product glulam into the two strength determining "materials" wood and finger joint separates the very complex problem into two smaller fields. The strength characteristics of each "material" may be determined accurately with the "Karlsruhe calculation model" as a function of the corresponding influencing parameters (strength of finger joints, wood properties...). The effect of each parameter on the strength properties of glulam beams may then be estimated numerically with the help of this statistical model.

A practible example:
The bending strength of glulam beams is governed by the tensile strength of the occuring finger joints and variations of finger joint strength values consequently affect the bearing capacity of glulam beams. Actually, nobody is able to estimate the effect of a 10% reduction or increase of finger joint strength on the bending strength of glulam beams.

On the basis of the statistical model, however, it is obvious that a variation of the strength values of finger joints only controls the "true" strength distribution of the "material" finger joint, i.e. the beams with failure due to finger joints, whereas the "true" strength distribution of "material" wood is not affected by this (with the assumption that wood quality was not changed). The "moving" of one distribution results in different ratios of the respective relevant strength values (5th-percentiles, mean values) and the effect on the bearing capacity of glulam beams may then be estimated with the help of the statistical model by using *fig. 2* and *fig. 3* for instance.

Hence this statistical model makes it possible to estimate the effect of varying strength determining parameters such as wood quality and strength of finger joints on the strength of glulam beams by "mixing" the expected "true" strength distributions of the beams with wood failure and finger joint failure.

## 3 Size effects

### 3.1 General

The "true" strength distributions of the two "materials" wood and finger joint may, as already mentioned, be calculated with the help of the "Karlsruhe calculation model".

According to the draft of Eurocode 5, design of glulam beams is based on the characteristic strength values (5th-percentiles) of a standard beam with a depth of 300 mm. If other beam sizes are used, modifications of these strength values are necessary.

Size effects have been the subject of many investigations, but most tests were performed either on small clear specimen (Bohannan 1966) or on solid timber (Newlin/Trayer 1924; Freas/Selbo 1954; Fewell/ Curry 1983; Madsen/ Buchannan 1984) with limited dimensions. Size effects on bending strength of glulam beams have not yet been investigated systematically with a significant number of tests. It is true that there have been numerous tests with glulam beams of different sizes, but other strength determining factors such as wood species, wood quality and strength of finger joints were not held constant. Furthermore some beams had no finger joints at all, others scarf joints. Hence with the existing test data it is hardly possible to clear up the question of size effects in glulam.

Within the "Karlsruhe calculation model", however, it is possible to keep constant all the marginal conditions and to vary only the dimensions of the beams. This makes it possible to investigate systematically size effects in glulam.

The simulation results of each beam were related to the strength values of the standard beam shown in *fig. 4*. This beam complies with the requirements of ISO 8375, and the corresponding characteristic strength values could be used as basic data in Eurocode 5. For this beam the length of the built-in boards (=distance between finger joints) was assumed to be 4,0 m on average with a standard deviation of 0,4 m.

For each variation of beam size, strength of finger joints and load configuration a total of 2000 simulations was performed. The simulation results were arranged in ascending order, and the $101^{th}$ value was chosen as the characteristic bending strength. This procedure guaranteed the accurate determination of the 5th-percentile, irrespective of the form of the strength distribution.

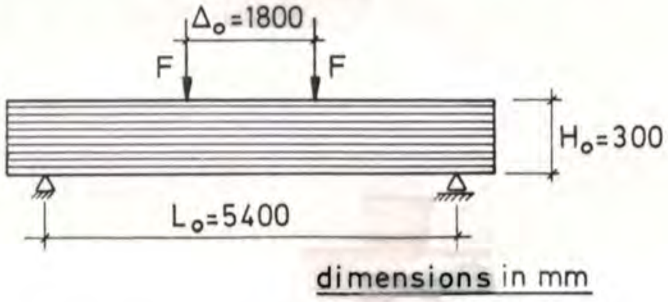



*fig.4 :* Standard beam

Apart from the 5th-percentile and mean value, the coefficient of variation of the strength distribution is needed by the statistical model. As the statistical model assumes the bending strength to be normally distributed, a normal distribution was "forced" through the calculated 5th-percentile $x_5$ and mean value m:

$$v = \frac{1 - x_5/m}{1{,}645} \quad . \tag{9}$$

Due to the fact that the essential strength values - 5th-percentile and mean value - have accurately been determined, this was thought to be a reasonable approximation.

## 3.2 Beams with failure due to finger joints

Starting from the strength values of the standard beam, the corresponding strength values of a beam with arbitrary dimensions may be calculated according to the following formulas:

$$x_{5,fj} = k_{L,5,fj} \cdot k_{H,5,fj} \cdot k_{F,5,fj} \cdot x^0_{5,fj} \tag{10a}$$

and

$$m_{fj} = k_{L,m,fj} \cdot k_{H,m,fj} \cdot k_{F,m,fj} \cdot m^0_{fj} \tag{10b}$$

where

$x_{5,fj}$ and $m_{fj}$ = 5th-percentile and mean value of bending strength of glulam beams with failure due to finger joints,

$x^0_{5,fj}$ and $m^0_{fj}$ = 5th-percentile and mean value of bending strength of the corresponding standard beam,

$k_{L,5,fj}$ and $k_{L,m,fj}$ = factors to describe the effect of beam length,

$k_{H,5,fj}$ and $k_{H,m,fj}$ = factors to describe the effect of beam depth,

$k_{F,5,fj}$ and $k_{F,m,fj}$ = factors to describe the effect of load configuration.

### 3.2.1 Effect of tensile strength of finger joints

The bending strength of glulam beams with failure due to finger joints is governed by the tensile strength of the finger joints occuring in the highly stressed regions of the beam (outer laminations). This dependency may be written as:

$$x^0_{5,fj} = k_{0,5,fj} \cdot f_{t,5,fj} \tag{11a}$$

and

$$m^0_{fj} = k_{0,m,fj} \cdot f_{t,m,fj} \tag{11b}$$

where

$x^0_{5,fj}$ and $m^0_{fj}$ = 5th-percentile and mean value of bending strength of the standard beam with failure due to finger joints,

$f_{t,5,fj}$ and $f_{t,m,fj}$ = 5th-percentile and mean value of tensile strength of finger joints,

$k_{0,5,fj}$ and $k_{0,m,fj}$ = factors to describe the effect of tensile strength of finger joints on the bending strength of the standard beam,

The course of the factors $k_{0,5,fj}$ and $k_{0,m,fj}$, depending on the coefficient of variation $v_{t,fj}$, is shown in *fig. 5*. It is essential to point out that these curves only are valid for tensile strength values, that have been determined with a test set up, which excludes any lateral displacement of the test specimen (i.e. not according to ISO 8375, c.f. Ehlbeck/ Colling 1986).

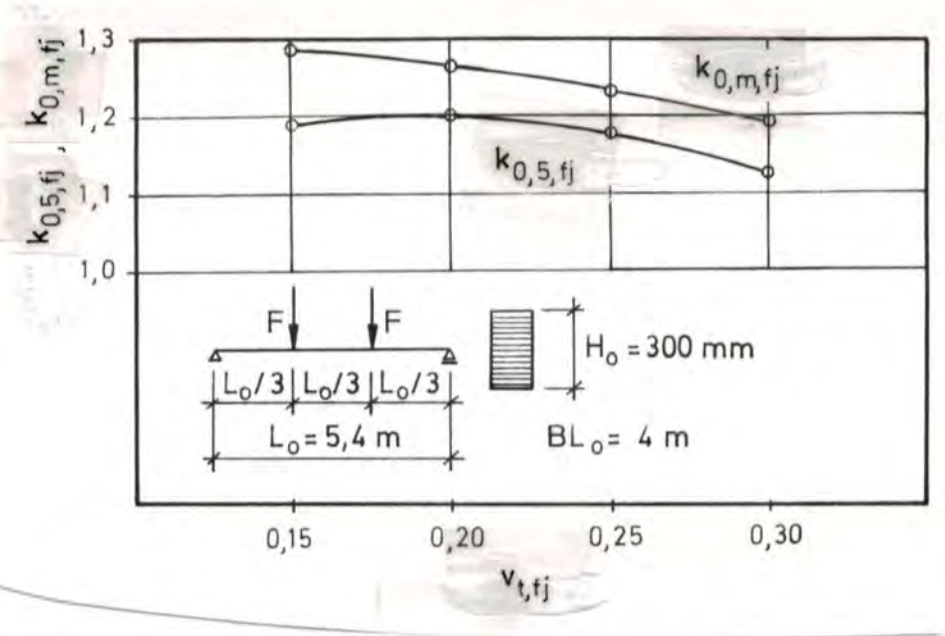



*fig. 5 :* Factors $k_{0,5,fj}$ and $k_{0,m,fj}$ to describe the effect of tensile strength of finger joints on the bending strength of the standard beam

According to Colling 1990a, a coefficient of variation of the tensile strength of finger joints of $v_{t,fj} \simeq 0{,}20$ is to be expected. Accepting this value, *fig. 5* shows that the characteristic bending strength of the standard beam is about 20% higher than the characteristic tensile strength of finger joints.

### 3.2.2 Length effect

Due to the more frequent occurence of finger joints, any increase of beam length and/or decrease of board lengths affects the bearing capacity of glulam beams. Therefore, length effect may be described with the following formulas:

$$k_{L,5,fj} = \left(\frac{L}{L_0} \cdot \frac{BL_0}{BL}\right)^{-\beta_{L,5,fj}} \qquad (12a)$$

and

$$k_{L,m,fj} = \left(\frac{L}{L_0} \cdot \frac{BL_0}{BL}\right)^{-\beta_{L,m,fj}} \qquad (12b)$$

where

| | |
|---|---|
| $L_0$, $BL_0$ = | length ($L_0$ = 5,4m) and average board length ($BL_0$ = 4m) of the standard beam, |
| L, BL = | length and average board length of an arbitrary beam, |
| $\beta_{L,5,fj}$ = and $\beta_{L,m,fj}$ | "Weibull" - exponents. |

The course of factors $k_{L,5,fj}$ and $k_{L,m,fj}$ is shown in *fig. 6*.

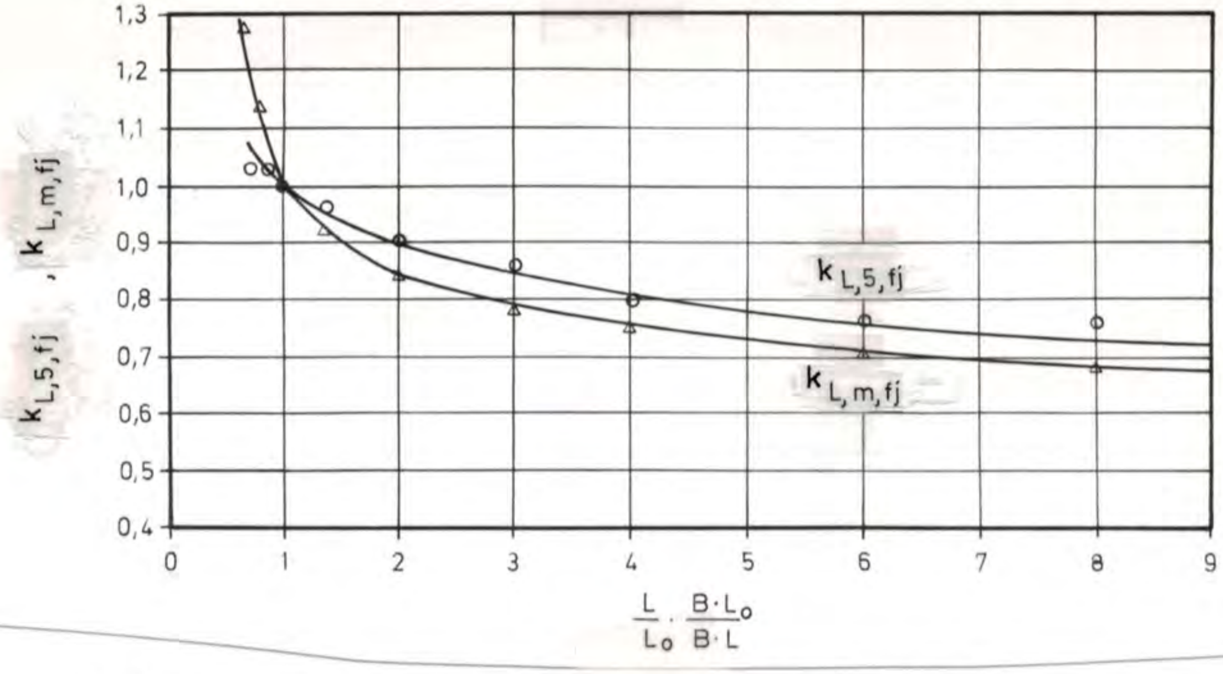



*fig. 6:* Factors $k_{L,5,fj}$ and $k_{L,m,fj}$ to describe length effects

In case of $k_{L,m,fj}$ an overproportional increase can be seen for values of $(L/L_0)\cdot(BL_0/BL) \leq 2$. This may be explained by the fact that in case of small beam lengths and/or large board lengths the probability of a finger joint occuring in the highly stressed zones is very small so that overproportional high strength values are possible. This results in a positive skewness of the strength distribution, affecting more the mean value than the 5th-percentile.

In case of $(L/L_0)\cdot(BL_0/BL) \geq 2$, the simulation results may be described with:

$$k_{L,m,fj} = 0{,}933\cdot\left(\frac{L}{L_0}\cdot\frac{BL_0}{BL}\right)^{-\beta_{L,m,fj}} \quad . \qquad (12c)$$

The adapted "Weibull" - exponents $\beta_{L,5,fj}$ and $\beta_{L,m,fj}$ are given in <u>*table 1*</u> in chapter 3.4.

### <u>3.2.3 Depth effect</u>

The factors $k_{H,5,fj}$ anf $k_{H,m,fj}$ describing depth effects may be calculated by:

$$k_{H,5,fj} = (\frac{H}{H_0})^{-\beta_{H,5,fj}} \tag{13a}$$

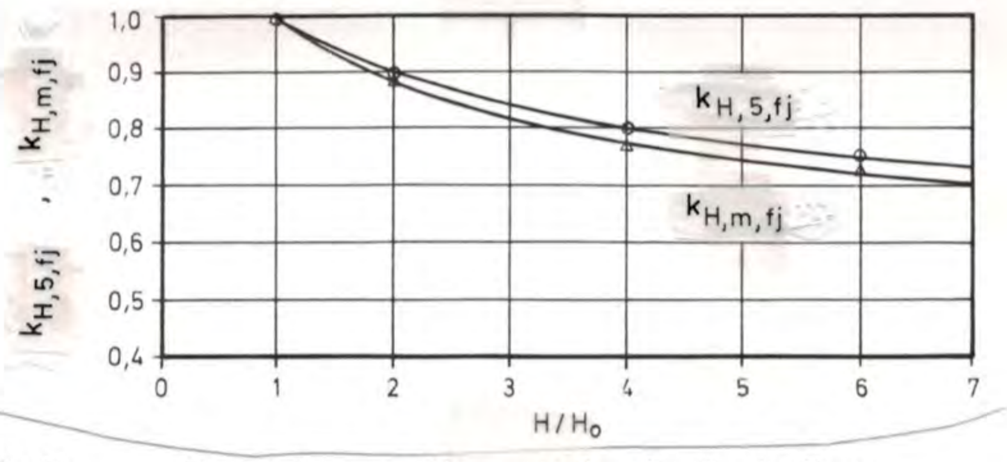
and

$$k_{H,m,fj} = (\frac{H}{H_0})^{-\beta_{H,m,fj}} \quad , \tag{13b}$$

where

$H_0$ = depth of the standard beam (H = 300 mm),
H = depth of an arbitrary beam.

The course of factors $k_{H,5,fj}$ and $k_{H,m,fj}$ is shown in <u>*fig. 7.*</u> Again, the curve for $k_{H,m,fj}$ is only valid for $(L/L_0)\cdot(BL_0/BL) \leq 2$.




<u>*fig. 7:*</u> Factors $k_{H,5,fj}$ and $k_{H,m,fj}$ to describe depth effects

The adapted "Weibull" - exponents are given in *table 1* (chapter 3.4).

### 3.2.4 Effect of load configuration

According to Weibull's theory of brittle fracture, the strength of a material depends on the size of stressed volume. This explains the fact that a beam with a single load has got a higher bearing capacity than a beam with third point loading.

In this paper, the effect of load configuration was investigated by varying the distance $\Delta$ between loads.

According to Colling 1986 this effect may be taken into account by

$$k_{F,5,fj} = \left(\frac{\beta_{F,5,fj} + \Delta/L}{\beta_{F,5,fj} + 1/3}\right)^{-\beta_{F,5,fj}} \tag{6a}$$

and

$$k_{F,m,fj} = \left(\frac{\beta_{F,m,fj} + \Delta/L}{\beta_{F,m,fj} + 1/3}\right)^{-\beta_{F,m,fj}} \quad . \tag{6b}$$

where

$\Delta/L$ = ratio of load distance and beam length of an arbitrary beam,
$1/3$ = ratio of load distance and beam length of the standard beam.

The course of the factors $k_{F,5,fj}$ and $k_{F,m,fj}$ is shown in *fig. 8*. Again, the curve for $k_{F,m,fj}$ is only valid for $(L/L_0)\cdot(BL_0/BL) \leq 2$.

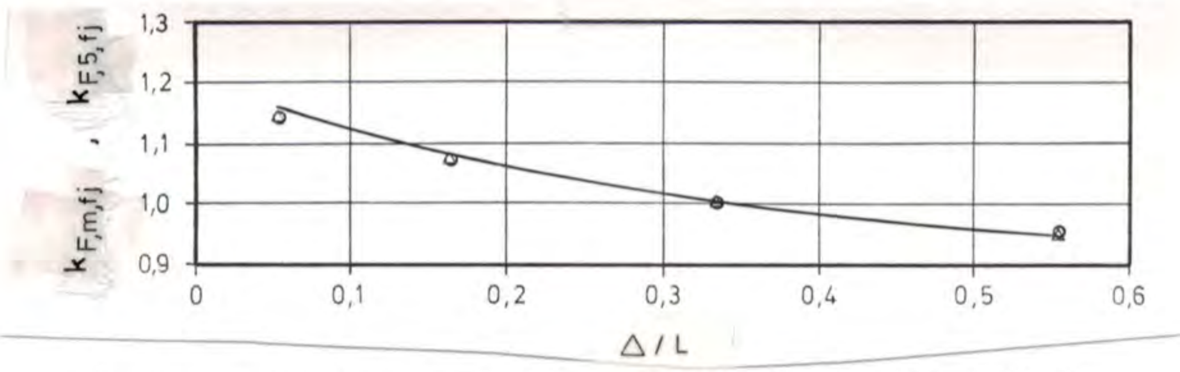



*fig. 8:* Factors $k_{F,5,fj}$ and $k_{F,m,fj}$ to describe the effect of load configuration

The adapted "Weibull" - exponents are again given in *table 1*.

## 3.3 Beams with wood failure

In chapter 3.2.1 the bending strength of glulam beams with failure due to finger joints was related to the tensile strength of finger joints. Therefore it would have been only logical to refer the strength of the standard beam with wood failure to the tensile strength of the laminations. This, however, leads to some problems:

- due to knots and accompanying grain deviations, lateral displacements do occur during a tensile test according to ISO 8375. These displacements cause additional bending stresses of the test specimen, resulting in a lower tensile strength. As these displacements are prevented when the laminations are part of a glulam beam, the so determined strength values do not apply to the conditions of a lamination in a glulam beam;
- in a glulam beam, longitudinal strains of weak zones (i.e. zones with knots and low MOE) are hindered by the adjacent laminations so that the arithmetic failure stress of the lamination in a glulam beam is higher than the corresponding tensile strength determined in a tension test (lamination effect);
- the tensile strength depends on the length of the test specimen (size effect). Due to the fact that a certain length of the specimen is needed to transmit the load from the grips into the test specimen and that furthermore the length of the built-in boards is not a constant, a tensile strength value, determined with a test specimen of a given length is not valid for all laminations.

In the further course of this paper, the bending strength of standard beams with wood failure will therefore directly be computed as a function of the wood properties KAR-value, wood density and MOE of the laminations, which are also used as criterions for wood grading.

In analogy to chapter 3.2, the bending strength values of a glulam beam with arbitrary dimensions may be calculated by:

$$x_{5,wood} = k_{L,5,wood} \cdot k_{H,5,wood} \cdot k_{F,5,wood} \cdot x^{0}_{5,wood} \qquad (14a)$$

and

$$m_{wood} = k_{L,m,wood} \cdot k_{H,m,wood} \cdot k_{F,m,wood} \cdot m^{0}_{wood} \quad . \qquad (14b)$$

where

| | | |
|---|---|---|
| $x_{5,wood}$ and $m_{wood}$ | = | 5th-percentile and mean value of bending strength of an arbitrary beam with wood failure |
| $x^0_{5,wood}$ and $m^0_{wood}$ | = | 5th-percentile and mean value of bending strength of the corresponding standard beam (depending on wood properties) |
| $k_{L,5,wood}$ and $k_{L,m,wood}$ | = | factors to describe the effect of beam length, |
| $k_{H,5,wood}$ and $k_{H,m,wood}$ | = | factors to describe the effect of beam depth, |
| $k_{F,5,wood}$ and $k_{F,m,wood}$ | = | factors to describe the effect of load configuration. |

Based on numerous simulations (Colling 1990a), the coefficient of variation of the bending strength of glulam beams with wood failure is expected to be $v^0_{wood} \simeq 0{,}11 - 0{,}16$. Therefore, size effects were investigated with the help of a "general" beam type whose laminations only had to meet the requirement $KAR \leq 0{,}5$. These beams comply with the strength class S10 (Güteklasse II) or better of DIN 4074. The bending strength of this beam type has got a coefficient of variation of 0,13 so that it was thought to be suited for the investigation of size effects.

### 3.3.1 Length effect

According to chapter 3.2.2, length effect may be described with

$$k_{L,5,wood} = \left(\frac{L}{L_0}\right)^{-\beta_{L,5,wood}} \qquad (15a)$$

and

$$k_{L,m,wood} = \left(\frac{L}{L_0}\right)^{-\beta_{L,m,wood}} \quad , \qquad (15b)$$

where $L_0$ corresponds to the length of standard beam ($L_0$ = 5,4m) and L to the length of an arbitrary beam.

The course of the factors $k_{L,5,wood}$ and $k_{L,m,wood}$ is shown in *fig. 9*.

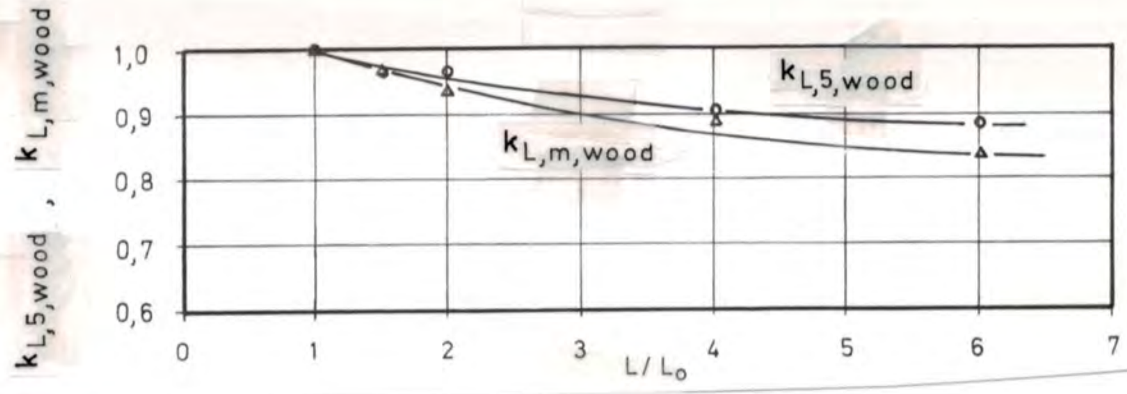



*fig. 9:* Factors $k_{L,5,wood}$ and $k_{L,m,wood}$ to describe length effects

The adapted "Weibull"-exponents are given in *table 2* of chapter 3.4.

### 3.3.2 Depth effect

Depth effects may be described with the following formulas:

$$k_{H,5,wood} = \left(\frac{H}{H_0}\right)^{-\beta_{H,5,wood}} \tag{16a}$$

and

$$k_{H,m,wood} = \left(\frac{H}{H_0}\right)^{-\beta_{H,m,wood}} , \tag{16b}$$

where $H_0$ is the depth of standard beam ($H_0$ = 300mm) and H the depth of an arbitrary beam.

The course of the factors $k_{H,5,wood}$ and $k_{H,m,wood}$ is shown in *fig. 10*.

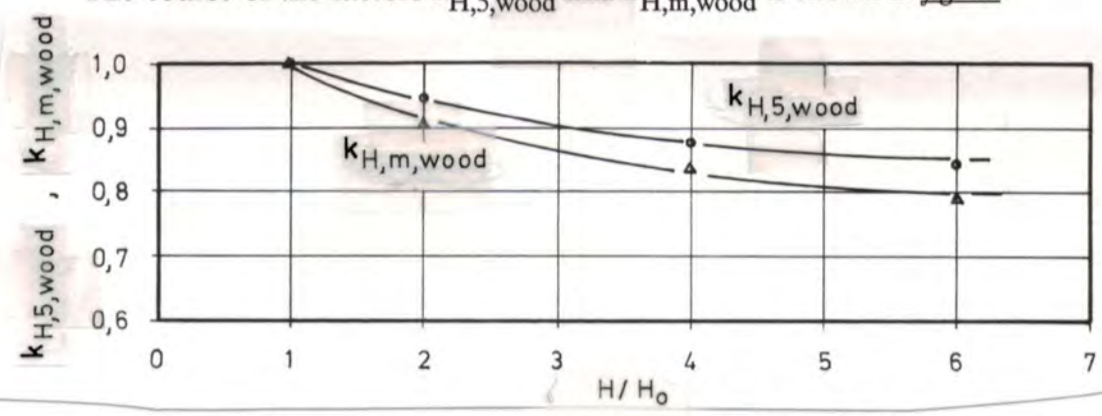



*fig. 10:* Factors $k_{H,5,wood}$ and $k_{H,m,wood}$ to describe depth effects

The adapted "Weibull" - exponents are again given in *table 2.*

### 3.3.3 Effect of load configuration

According to *eq. (13)*, the factors $k_{F,5,wood}$ and $k_{F,m,wood}$ may be calculated by

$$k_{F,5,wood} = \left(\frac{\beta_{F,5,wood} + \Delta/L}{\beta_{F,5,wood} + 1/3}\right)^{-\beta_{F,5,wood}} \qquad (17a)$$

and

$$k_{F,m,wood} = \left(\frac{\beta_{F,m,wood} + \Delta/L}{\beta_{F,m,wood} + 1/3}\right)^{-\beta_{F,m,wood}} , \qquad (17b)$$

where

$\Delta/L$ = ratio of load distance and beam length of an arbitrary beam,
$1/3$ = ratio of load distance and beam length of the standard beam (third point loading).

The course of these factors is shown in *fig. 11*.

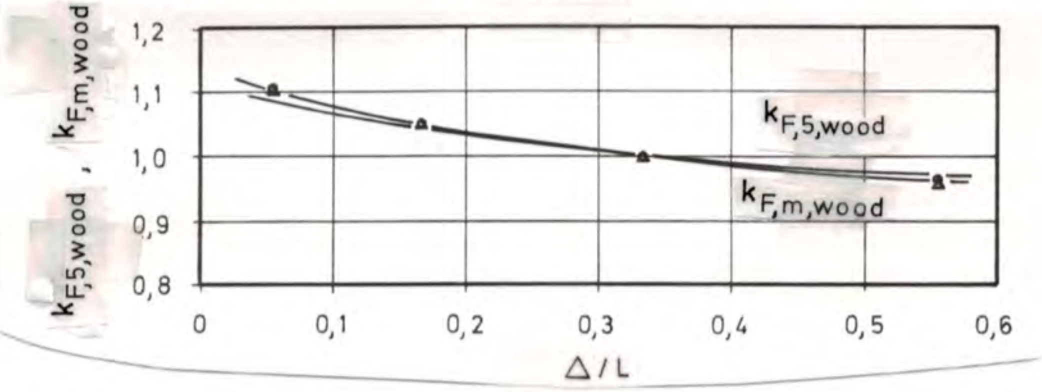



*fig. 11:* Factors $k_{F,5,wood}$ and $k_{F,m,wood}$ to describe the effect of load configuration

The adapted "Weibull" - exponents are given in *table 2.*

## 3.4 Discussion

The "Weibull" - exponents which fit best the simulation results are summarized in *table 1* and *table 2*.

*table 1*: "Weibull" - exponents to describe size effects; beams with failure due to finger joints

| $\beta_{L,5,fj}$ | $\beta_{H,5,fj}$ | $\beta_{F,5,fj}$ | $\beta_{L,m,fj}$ | $\beta_{H,m,fj}$ | $\beta_{F,m,fj}$ |
|---|---|---|---|---|---|
| 0,15 | 0,16 | 0,15 | 0,15 | 0,18 | 0,15 |

*table 2:* "Weibull" - exponents to describe size effects; beams with wood failure

| $\beta_{L,5,wood}$ | $\beta_{H,5,wood}$ | $\beta_{F,5,wood}$ | $\beta_{L,m,wood}$ | $\beta_{H,m,wood}$ | $\beta_{F,m,wood}$ |
|---|---|---|---|---|---|
| 0,07 | 0,09 | 0,07 | 0,10 | 0,13 | 0,10 |

The following tendencies may be stated :

- size effects are more pronounced in case of beams with failure due to finger joints than in case of beams with wood failure; this may be explained by the higher variability of strength data;
- length effect may be described with the same "Weibull"-exponents as the effect of load configuration;
- depth effects are more pronounced than length effects or the effect of load configuration,
- in case of beams with finger joint failure, size effects on the characteristic bending strength may in good approximation be described with one single exponent $\beta_{5,fj}$ = 0,15.

Based on the bending strength values 5th-percentile and mean value of the standard beams, these investigations make it possible to calculate the corresponding strength values of "materials" wood and finger joint with arbitrary dimensions by using the "mixing procedure" of chapter 2.

# 4 Verification of the statistical model by bending tests

## 4.1 Beam tests

The reliability of the statistical model was to be verified by bending tests. A total of 42 bending tests was performed. Beam dimensions and test set-up as shown in *fig. 12* were not identical with the corresponding values of the standard beam (cf. *fig. 4*). Hence, a verification of the factors describing size effects was also possible.

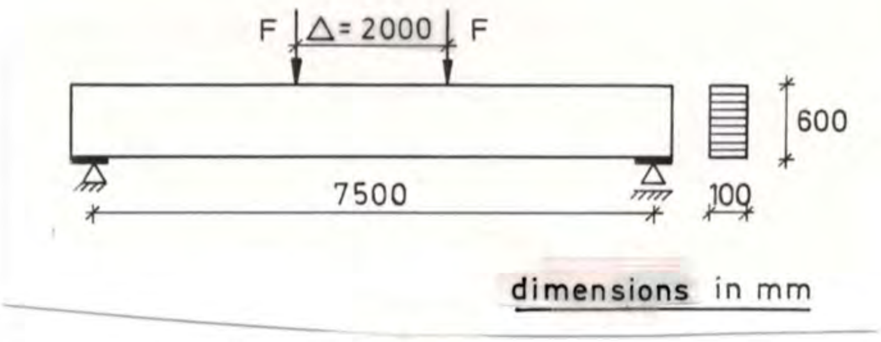



*fig. 12:* Beam dimensions and test set-up

Six test series with seven replications were performed. The outer three laminations of each beam had to meet the requirements given in *table 3*.

*table 3:* Requirements raised to the wood properties of the three outer laminations on both sides

| test series | requirements |
|---|---|
| I | 0,35 ≤ KAR |
| II | 0,20 ≤ KAR ≤ 0,35 |
| III | KAR ≤ 0,20 |
| IV | 500 kg/m³ ≤ ρ [1] |
| V | 15000 N/mm² ≤ E |
| VI | 15000 N/mm² ≤ E und KAR ≤ 0,20 |

[1] density at u = 12 - 14%

The beams of the first three test series were built up by using only visual criterions, whereas the outer laminations of the other beam types were sorted according to a possible machine grading (series IV and V) or a combined visual/machine grading (series VI).

MOE of the boards was determined according to Görlacher 1990 by using a method of longitudinal vibrations.

Due to the fact that half of the beams were systematically built up with laminations having different MOE (series IV-VI), the ultimate bending stresses were calculated according to the theory of transformed sections:

$$\mathrm{ef}\,\sigma_B = \frac{\max M}{\mathrm{ef\,EI}} \cdot E_0 \cdot H/2, \tag{18}$$

where

max M = bending moment at failure,
$E_0$ = MOE of the outermost lamella on tension side,
H = beam depth (H = 600 mm),
ef EI = $\sum E_i I_i + \sum E_i A_i a^2_i$
= effective bending resistance, taking into account the different MOE - values of the laminations.

Test results including failure modes are given in <u>*table 4*</u>.

## 4.2 Calculation results and predictions of the statistical model

For each test series, the strength characteristics of standard beams (with wood failure and failure due to finger joints) were calculated by the "Karlsruhe calculation model".

Comparative tests with finger joints showed that on production day of glulam beams, the strength of finger joints was about 10% higher than on average. This was taken into account during the calculations.

*table 4 :* Test results

| beam | ef $\sigma_B$ N/mm² | failure mode | beam | ef $\sigma_B$ N/mm² | failure mode |
|---|---|---|---|---|---|
| I - 1 | 32,9 | knot | IV - 1 | 43,1 | fj [1] |
| I - 2 | 37,5 | knot | IV - 2 | 47,7 | knot |
| I - 3 | 33,7 | knot | IV - 3 | 35,4 | fj |
| I - 4 | 34,7 | knot | IV - 4 | 50,6 | fj |
| I - 5 | 35,3 | knot | IV - 5 | 45,7 | |
| I - 6 | 30,7 | knot | IV - 6 | 51,9 | shear failure |
| I - 7 | 38,3 | knot | IV - 7 | 47,6 | knot |
| mean value | 34,7 | | mean value | 46,0 | |
| coeff. of var. | 0,08 | | coeff. of var. | 0,12 | |
| II - 1 | 32,2 | knot | V - 1 | 37,1 | knot |
| II - 2 | 35,5 | knot | V - 2 | 49,5 | fj |
| II - 3 | 41,8 | fj[1] | V - 3 | 43,4 | knot |
| II - 4 | 45,7 | fj | V - 4 | 49,3 | knot |
| II - 5 | 44,1 | knot | V - 5 | 40,9 | fj |
| II - 6 | 33,6 | fj | V - 6 | 48,5 | fj |
| II - 7 | 39,6 | fj | V - 7 | 65,0 | fj |
| mean value | 38,9 | | mean value | 47,7 | |
| coeff. of var. | 0,15 | | coeff. of var. | 0,19 | |
| III - 1 | 42,8 | fj | VI - 1 | 53,3 | knot |
| III - 2 | 37,7 | fj | VI - 2 | 39,8 | knot |
| III - 3 | 36,4 | fj | VI - 3 | 48,7 | knot |
| III - 4 | 32,3 | knot | VI - 4 | 45,4 | fj |
| III - 5 | 41,3 | fj | VI - 5 | 49,3 | fj |
| III - 6 | 44,7 | knot | VI - 6 | 54,6 | knot |
| III - 7 | 47,1 | fj | VI - 7 | 60,2 | fj |
| mean value | 40,3 | | mean value | 50,2 | |
| coeff. of var. | 0,13 | | coeff. of var. | 0,13 | |

[1] fj = finger joint

The length of the built-in boards was 4 m on average, so that size effects could be computed with the following values:

$$(L/L_0)\cdot(BL_0/BL) = (7500/5400)\cdot(4/4) = 1{,}39 = L/L_0$$

$$H/H_0 = 600/300 = 2{,}0$$

$$\Delta/L = 2000/7500 = 0{,}267 \quad .$$

Assuming that <u>all</u> beams fail due to a finger joint failure the expected bending strength values $x_{5,fj}$ and $m_{fj}$ are given in *table 5a and b*. In case of mean value, these factors were calculated according to Colling 1990a, because the corresponding equations in chapter 2 are only valid for $(L/L_0)\cdot(BL_0/BL) \geq 2$.

*table 5a:* Calculation of 5th-percentiles $x_{5,fj}$

| test series | $x^0_{5,fj}$ N/mm² | $k_{L,5,fj}$ | $k_{H,5,fj}$ | $k_{F,5,fj}$ | $x_{5,fj}$ N/mm² |
|---|---|---|---|---|---|
| I - III | 30,8 | | | | 26,9 |
| IV | 37,0 | 0,952 | 0,895 | 1,022 | 32,2 |
| V - VI | 40,4 | | | | 35,2 |

*table 5b:* Calculation of mean values $m_{fj}$

| test series | $m^0_{fj}$ N/mm² | $k_{L,m,fj}$ | $k_{H,m,fj}$ | $k_{F,m,fj}$ | $m_{fj}$ N/mm² |
|---|---|---|---|---|---|
| I - III | 48,9 | | | | 40,4 |
| IV | 59,1 | 0,921 | 0,864 | 1,037 | 48,8 |
| V - VI | 61,1 | | | | 50,4 |

Assuming that <u>all</u> beams fail in the area of knots (wood failure), the expected bending strength values $x_{5,wood}$ and $m_{wood}$ are given in *table 6a and b*.

*table 6a :* Calculation of 5th-percentiles $x_{5,wood}$

| test series | $x^0_{5,wood}$ N/mm² | $k_{L,5,wood}$ | $k_{H,5,wood}$ | $k_{F,5,wood}$ | $x_{5,wood}$ N/mm² |
|---|---|---|---|---|---|
| I | 28,4 | | | | 26,4 |
| II | 33,3 | | | | 31,0 |
| III | 37,3 | | | | 34,7 |
| | | 0,977 | 0,940 | 1,013 | |
| IV | 40,9 | | | | 38,1 |
| V | 43,0 | | | | 40,0 |
| VI | 47,4 | | | | 44,1 |

*table 6b :* Calculation of mean values $m_{wood}$

| test series | $m^0_{wood}$ N/mm² | $k_{L,m,wood}$ | $k_{H,m,wood}$ | $k_{F,m,wood}$ | $m_{wood}$ N/mm² |
|---|---|---|---|---|---|
| I | 39,1 | | | | 35,2 |
| II | 43,7 | | | | 39,3 |
| III | 47,9 | | | | 43,1 |
| | | 0,968 | 0,914 | 1,017 | |
| IV | 52,0 | | | | 46,8 |
| V | 54,2 | | | | 48,7 |
| VI | 58,2 | | | | 52,3 |

Tests with normal glulam beams, however, neither always fail due to finger joints nor always fail in the area of knots. Knowing the strength distributions of the two "materials" wood and finger joint, the bending strength of the final product glulam may be estimated by the "mixing procedure" of the statistical model.

For each test series, the expected 5th-percentile $\sigma_5$ and the corresponding median value $\sigma_{50}$ are given in <u>table 7</u>. In all cases, except series I, "material" finger joint was found to be the weaker "material" noted 1, so that the wanted bending strength values of test beams could be determined according to *fig. 2* and *fig. 3* (with $v_1 \simeq 0,2$). In case of series I, the strength values were calculated according to Colling 1990a (with $v_1 \simeq 0,15$).

*<u>table 7 :</u>* Calculation of 5th-percentiler $\sigma_5$ and of median values $\sigma_{50}$ of bending strength

| series | $\frac{x_{5,1}}{x_{5,2}}$ | $\frac{m_1}{m_2}$ | $v_1$ | $\frac{\sigma_5}{x_{5,1}}$ | $\frac{\sigma_{50}}{m_1}$ | $\sigma_5$ N/mm$^2$ | $\sigma_{50}$ N/mm$^2$ | $m^1_{test}$ N/mm$^2$ |
|---|---|---|---|---|---|---|---|---|
| I | 0,981 | 0,871 | 0,15 | 0,933 | 0,953 | 24,6 | 33,5 | 34,7 |
| II | 0,868 | 1,028 | 0,20 | 0,985 | 0,900 | 26,5 | 36,4 | 38,9 |
| III | 0,775 | 0,937 | 0,20 | 0,998 | 0,949 | 26,8 | 38,3 | 40,3 |
| IV | 0,845 | 1,043 | 0,20 | 0,991 | 0,893 | 31,9 | 43,6 | 46,0 |
| V | 0,880 | 1,035 | 0,20 | 0,980 | 0,900 | 34,5 | 45,4 | 47,7 |
| VI | 0,798 | 0,964 | 0,20 | 1,000 | 0,940 | 35,2 | 47,4 | 50,2 |

[1] mean values of *table 4*

Furthermore the probability $P_{fj}$ of a finger joint failure was calculated for each beam type according to *eq.(9)* (see <u>table 8</u>).

*<u>table 8 :</u>* Expected probability $P_{fj}$ of a finger joint failure

| test series | $\frac{m_{wood} - m_{fj}}{\sqrt{s^2_{wood} + s^2_{fj}}}$ | $P_{fj}$ % |
|---|---|---|
| I | -0,539 | 30 |
| II | -0,115 | 45 |
| III | 0,281 | 61 |
| IV | -0,174 | 43 |
| V | -0,234 | 40 |
| VI | 0,090 | 54 |

## 4.3 Discussion

A comparison between test results and calculation results is shown in *fig. 13* and a very good agreement can be seen.

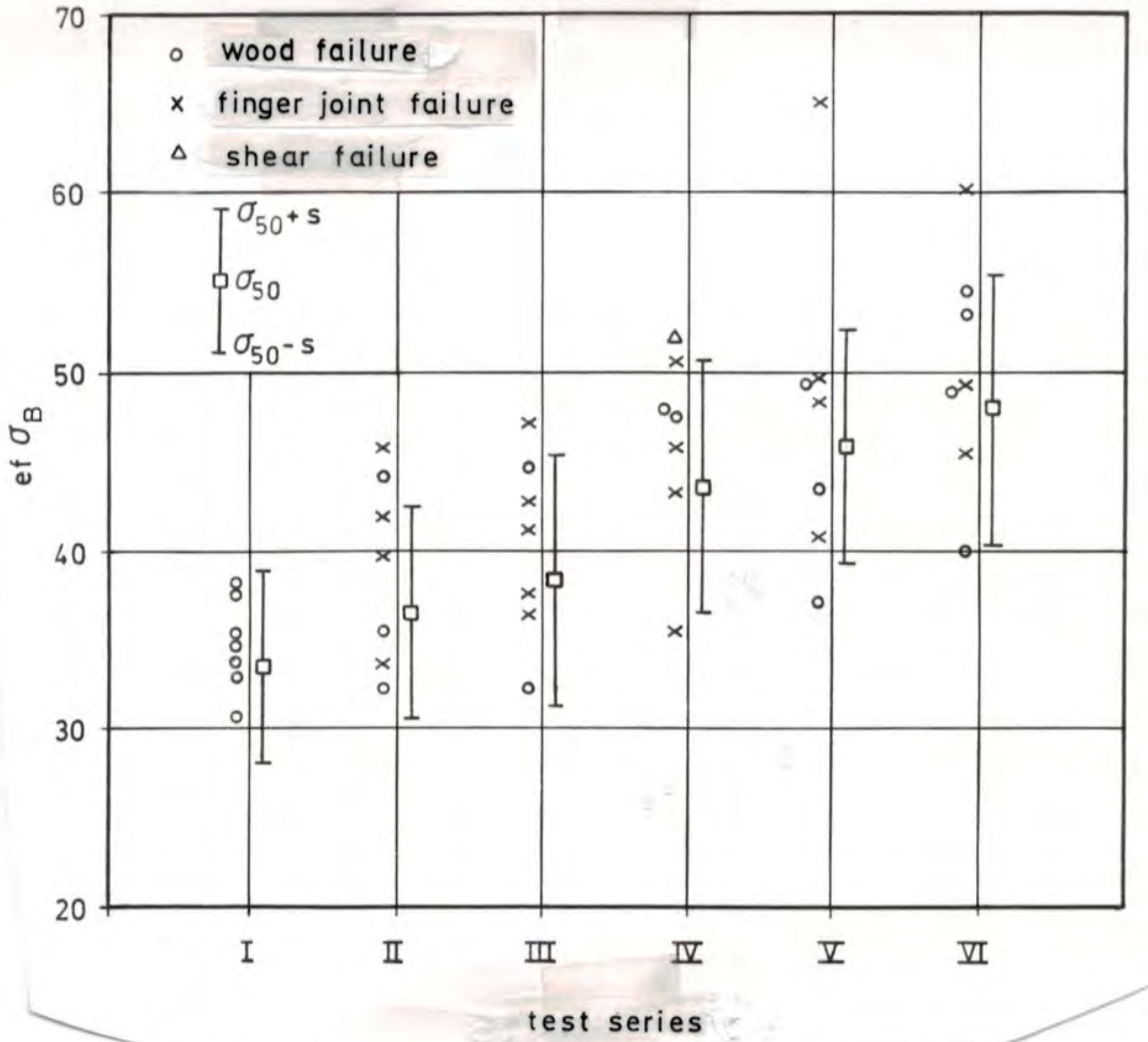



*fig. 13:* Comparison between test results and calculation results

The following concurrent tendencies may be stated:

- a stronger visual grading leads to a more frequent finger joint failure (cf. series I-III). This may be explained by the fact that lower KAR-values do increase the strength of the laminations themselves, whereas the strength of finger joints is not affected by this. This leads to a greater distance between the strength distributions of the two "materials" resulting in the fact that the final product glulam orientates itself stronger by the weaker "material";

- if requirements are raised to the density and/or MOE of the laminations, higher bending strength values are possible than in case of a pure visual grading. This may be explained by the fact that a higher density or MOE not only leads to a higher strength of the laminations, but also to a higher strength of finger joints. In this case, the strength distributions of both "materials" are moved in the direction of higher strength values.

Thus, these tests showed that, based on the calculation results of the "Karlsruhe calculation model", the statistical model is able to describe the behaviour of glulam beams as a function of the two strength determining parameters strength of finger joints and wood quality.

In contrast with the test results, the calculation results allow some statements about the expected characteristic bending strength values of the test beams:

- *Table 7* shows that the difference between the 5th-percentiles of series II and III is less than the difference between the corresponding median values. This indicates that a more severe visual grading only partly leads to higher characteristic bending strength values;
- a distinct increase of the 5th-percentile is only possible with requirements raised to the density or MOE of the laminations;
- with the exception of series I, the expected characteristic bending strength of every beam type was found to be approximately equal to the 5th-percentile of bending strength of beams with failure due to finger joints. This indicates that in cases of medium and high strength beams, the characteristic bending strength will solely be controlled by the strength properties of finger joints.

## 5 Summary and outlook

A statistical model was presented which divides the totality of glulam beams into two groups: beams with wood failure and beams with finger joint failure. Based on the "true" strength distributions of these two groups, it is possible to compute the strength characteristics of the final product glulam depending on wood properties and the strength of finger joints.

Due to the fact that the "true" strength distributions can hardly be determined by tests, the required strength distributions may be calculated with the "Karlsruhe calculation model".

Tests with glulam beams showed a very good agreement between test results and calculation results.

Numerous Monte Carlo simulations showed that in many (most) cases finger joints are the factors controlling the characteristic bending strength of glulam beams.

Furthermore, investigations about the effect of beam size and load configuration showed that size effects are more pronounced in case of beams with finger joint failure, a tendency which may be explained by the higher variability of strength data of finger joints. This indicates that the dominant influence of finger joints even increases with increasing beam size.

Hence, high strength beams can only be produced with high strength finger joints and if it is planned to achieve a specific characteristic bending strength of glulam beams, the finger joints must meet certain strength requirements.

In this connection, the tensile strength of finger joints is the decisive factor, but as tensile strength can hardly be determined by the glulam factories in the course of a current quality control, the desired strength values must be estimated by the corresponding bending strength. Knowing the ratio between tensile and bending strength of finger joints, it should consequently be possible to estimate the strength of glulam beams. This ratio is being investigated in the current research project, mentioned in chapter 1.

The splitting up of the final product glulam into the two strength determining factors wood and finger joint made it thus possible to clearly identify the dominant influence of the finger joints upon the bending strength of glulam beams. Recent investigations about the strength of finger joints (Ehlbeck/Colling 1990) indicate, that it will only be possible to achieve high strength values for finger joints and consequently also for glulam beams by using a machine grading which is based on density and/or MOE of the laminations.

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