Warning: Set theory can cause perceptual problems!

Scrooge McDuck every day receives 10 $ and issues 1 $. Since a comic character lives forever his wealth grows immeasurably. However if he issues always the dollars received first and if he applies modern set theory, then he will go bankrupt. [1, 2]

The little demon resides in his home with two rooms. In the morning he leaves his bedroom for his living room, and in the evening he returns to his bedroom. This continues in eternity because demons are immortal. There is no problem, no paradox and no "final result". A similar story, often misunderstood as a paradox, has been told by Thomson already. [3] However, if we apply set theory, then the number of the little demon's returns to the bedroom will become exhausted. [4] His bedroom will remain empty. Alas the number of returns to the living room will become exhausted too, so the living room will remain empty. The little demon has diappeared. (Note that the sequence of *days* of the little demon is monotonically increasing. So there is no excuse with non-monotonicity.)

These results are due to the exhaustibility of infinite sets, the so-called set-limit [5], which is the foundation of transfinite set theory which in turn is considered as the foundation of modern mathematics by most mathematicians: Those many which are not experts in this field simply believe in the expertise of the experts. These comparatively few experts have to accept exhaustibility because otherwise, by the relativistic equivalence of temporal axis and spatial "real" axis, the idea of "countable set" becomes untenable – and in addition a great deal of their life's work.

Obviously these results are nonsensical. But how could this credo-in-absurdum-attitude evolve? A special section in the brain must have become infected during the study of set theory. Apparently this infect does not impair intelligence but it prevents any perceptibility of the ridicule of the situation.

I am committed to prevent newbies from this bad influence. As experience shows the chances are good if they are warned in time. I conclude this from hundreds of my own students who with rational scepticism refuse to accept these "results". They simply know the mathematical limit of McDuck's wealth: It is the (improper) analytical limit of the sequence $(9n)_{n\in\mathbb{N}}$, namely infinity ∞ – and nothing else.

Therefore I would like to distribute this text as widely as possible.

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References

[1] W. Mückenheim: Transfinity - A Source Book (2017) p. 252

[2] A. Fraenkel, a leading proponent of set theory writes: "Well known is the story of Tristram Shandy who undertakes to write his biography, in fact so pedantically, that the description of each day takes him a full year. Of course he will never get ready if continuing that way. But if he would live infinitely long (for instance a 'countable infinity' of years), then his biography would get 'ready', because every day in his life, how late ever, finally would get its description. No part of his biography would remain unwritten, for to each day of his life a year devoted to that day's description would correspond." [A.A. Fraenkel: "Einleitung in die Mengenlehre" 3rd ed., Springer, Berlin (1928) p. 24]

[3] J.F. Thomson: "Tasks and super-tasks", Analysis 15 (1954) 1-13
Wikipedia: Thomson's lamp

[4] W. Mückenheim: Transfinity - A Source Book (2017) p. 21

[5] A sequence (M_n) of sets M_n has a limit Lim M_n if and only if

$$\text{Lim}\, M_n = \text{LimSup}\, M_n = \text{LimInf}\, M_n$$

where

$$\text{LimSup}\, M_n = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} M_k$$

$$\text{LimInf}\, M_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} M_k\,.$$

[S.I. Resnick: "A probability path", Birkhäuser, Boston (1998) p. 6]
Wikipedia: Limit superior and limit inferior

Information about the author

Wikipedia: https://de.wikipedia.org/wiki/Wolfgang_Mückenheim

Homepage: https://www.hs-augsburg.de/~mueckenh/

W. Mückenheim: "Die Geschichte des Unendlichen", 6th ed., Maro, Augsburg (2011)

W. Mückenheim: "Mathematik für die ersten Semester", 4th ed., De Gruyter, Berlin (2015)